Regular Article

Self-organized criticality: a new approach to multihadron production[*]

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Abstract. We apply the concept of self-organized criticality in statistical physics to the study of multihadron production in high energy collisions.

1 Introduction

In the last half of the past century, the central issues of elementary particle physics were

- what are the ultimate consituents of matter?

- what are the forces between them?

- what is the ultimate theory: QCD, electroweak, standard model, GUT, TOE?

They reflect the age-old reductionist approach to the study of matter.

The transition to the new millenium witnessed, in various degrees of clarity, a change of paradigm. It can perhaps be best summarized in the words of Per Bak [1]

The laws of physics are simple, but nature is complex.

Bak went on to ask

How can the universe start with a few types of elementary particles at the big bang, and end up with life, history, economics and literature? Why did the big bang not form a simple gas of particles or condense into one big crystal?

In other words, the new issue was to understand how the structured complexity of the world around us could arise. Thus, the new concepts determining much of the work of the past twenty years are

- emergence, complexity, fractality, chaos

- non-equilibrium behavior, self-organization

In physics, this has led to more intensive studies of emergent phenomena in non-equilibrium processes, in mathematics to that of fractal structures. It has moreover led to a general framework applicable as well to swarm formation in biology and to financial fluctuations in market patterns. In this talk, I want to show how it can provide a new view of multihadron production in high energy collisions [2].

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2 Criticality

Let us begin by recalling the standard study of correlations in a system of identical constituents (spins, particles, birds,...) subject to next neighbor interaction. For a given value of the control parameter T ("temperature") the correlation of two constituents at a separation r is given by the correlation function

$$\Gamma(r,T) \sim \frac{a}{r^p} \exp -r/\lambda, \tag{1}$$

in terms of a dimensional constant a, an emergent correlation length $\lambda(T)$ and a power-law exponent $p \simeq 1$. The correlation is thus scale-dependent, it becomes weaker with increasing separation,

$$\frac{\Gamma(2r,T)}{\Gamma(r,T)} = (1/2)^p \exp -r/\lambda, \tag{2}$$

measured in units of the correlation length. At the critical point of the system, the correlation length diverges $\lambda \to \infty$, so that

$$\Gamma(r,T_c) \sim \frac{a}{r^p} \tag{3}$$

and hence the relative correlation for two different separations becomes scale-independent,

$$\frac{\Gamma(2r,T_c)}{\Gamma(r,T_c)} = (1/2)^p. \tag{4}$$

The correlation is now independent of the separation r, there is no longer any self-organized scale λ.

3 Self-organized criticality

For systems in equilibrium, we have a control parameter T and an order parameter $m(T)$. To achieve criticality, an outside operator tunes the temperature adiabatically, $T \to T_c$, and at T_c, $m(T)$ changes abruptly. In other words, tuning of the control parameter changes the order parameter.

Non-equilibrium systems, on the other hand, evolve on their own, there is no tunig operator. As a result, the order parameter changes. Given suitable dynamics the evolution drives the system to a critical point ("critical attractor"). We now have an evolving order parameter, which results in a changing control parameter.

Per Bak has illustrated this in his now well-known sand-pile scenario [3] (see Fig. 1). Pouring sand slowly onto a flat surface leads to the formation of a sand pile, whose slope G continues to increase. When it reaches a critical value G_c, avalanches descend, and as more sand is added, more avalanches occur. We now record in the course of time the number of avalanches of size s, with the size determined by the amount of sand moved. The result is

$$n(s) = \left(\frac{a}{s}\right)^p \; \to \; \log n(s) = p \log s + \text{const.}; \quad \frac{n(s)}{n(2s)} = (1/2)^p \neq f(s). \tag{5}$$
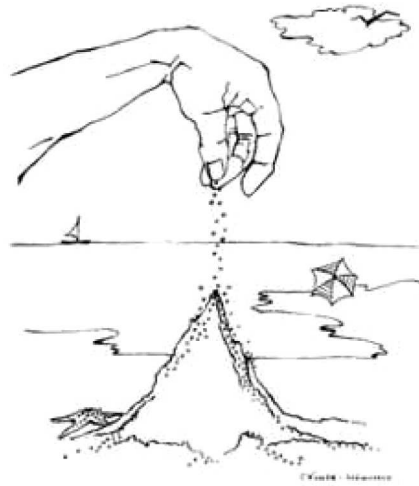
Fig. 1. The Sand-Pile scenario.
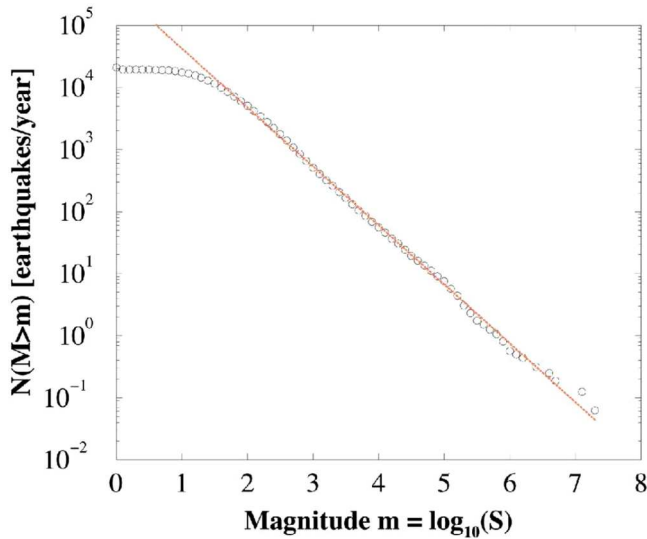
Fig. 2. Earthquake distribution in New Mexico.

We thus have a constant input, the pouring of sand, driving the slope to criticality; the output then concists of avalanches with a power-law size distribution, leading to scale-invariant size ratios.

Another application is given by the study of earthquakes. Measurements in New Mexico (see Fig. 2) give for the number $n(s)$ of earthquakes of size s on the Richter scale the form (see Ref. [1]),

$$\log n(s) = p \log s + \text{const.} \qquad (6)$$

Here the input is the increasing pressure in the crust of the earth, the resulting output the number of earthquakes of size s, in the form of a power-law in s.
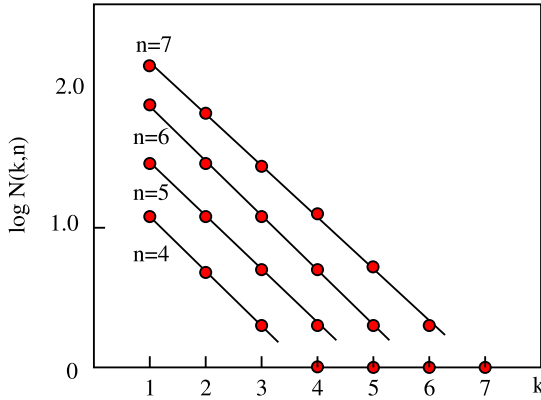
Fig. 3. Partition entropy $S(k, n)$ vs. k for different n.

The power-law form is seen to be quite well satisfied over six orders of magnitude. The deviations at low s arise from difficulties in detecting very weak earthquakes.

4 Scale-invariance

A simple example of scale-invariance is given by the ordered partitioning of an integer n into integers [4], such as

$$n = 3 : 3, 2 + 1, 1 + 2, 1 + 1 + 1 \qquad q(3) = 4, \tag{7}$$

for $n = 3$, there are $q(n = 3) = 4$ partitions. It is readily seen that the general result is

$$q(n) = 2^{n-1} = \frac{1}{2} \exp\{n \ln 2\}. \tag{8}$$

Note that the unordered case is more difficult, see Hardy and Ramanujan. We now ask how often the number k occurs in the set of all partitionings of n; we then want to define this number $N(k, n)$ in terms of a strength $s(k)$ of the number k. It seems natural to define this strength as the number of partionings of k itself,

$$s(k) = q(k) = \frac{1}{2} \exp(k \ln 2). \tag{9}$$

In a framework of self-organized criticality, we then expect the number $N(s(k), n)$ of partionings of strength $s(k)$ to show power-law behavior,

$$N(s(k), n) = \alpha(n)[s(k)]^{-p}. \tag{10}$$

For the partion entropy $S(k, n) = \log N(s(k), n)$ we then obtain the form

$$S(k, n) = \log N(k, n) = -k(p \log e \ln 2) + \text{const.}(n,). \tag{11}$$

In Figure 3, this is already seen to be well-satisfied for quite small values of n.

An integer consists of integers, which consist of integers,...:that brings to mind Rolf Hagedorn, who postulated that "fireballs consist of fireballs, which consist of

fireballs,...". Here we have once more a partitioning problem, but a generalized one, in which fireballs consist of moving fireballs, having kinetic energy, which has to be taken into account in the partitioning.

The equation governing the composition of fireballs is Hagedorn's statistical bootstrap equation [5], stating that there are $\rho(m)$ states of mass m

$$\rho(m) = \delta(m-m_0) + \sum_N \frac{1}{N!} \left[\frac{4\pi}{3(2\pi m_0)^3} \right]^{N-1} \int \prod_{i=1}^N [dm_i \; \rho(m_i) \; d^3 p_i] \; \delta^4(\Sigma_i p_i - p),$$

(12)

with m_0 for the ground state hadron. Its solution is given by [6]

$$\rho(m) \sim [1 + (m/\mu_0)]^{-a} \epsilon^{m/T_H} \; \to \; \ln \rho \sim \frac{m}{T_H} - a \ln[1 + (m/\mu_0)],$$

(13)

with a normalization constant μ_0. The crucial constant determining the exponential growth, the so-called Hagedorn "temperature" T_H, is the solution of the equation

$$\left(\frac{2}{3\pi} \right) \left(\frac{T_H}{m_0} \right) K_2(m_0/T_H) = 2 \ln 2 - 1.$$

(14)

Note that this Hagedorn "temperature" is totally of combinatoric origin; it corresponds to the "temperature" $\Theta = 1/\ln 2$ in the partitioning of integers. It becomes a temperature only in the partition function of a resoance gas of states with spectral weights $\rho(m)$.

5 High energy hadron production

The aim now is to formulate hadron production in high energy collisions as self-organized criticality [2]. The initial state just after the collision is a beam of energetic colored partons flying along the collision axis. In their passage, they lose energy by doing work against the physical vauum: the collision is a non-equilibrium process. The passage sorts the partons: at small rapidity, we have the slowest, and with increasing rapidity the faster and faster ones. This corresponds to the well-known "inside-outside" cascade of Bjorken [7].

In the conventional scenario describing such collisions [8,9], one considers a slice at fixed longitudinal rapidity, corresponding to a partial system at fixed time. It is assumed that the resulting system is a bubble of deconfined medium (QGP) in full local equilibrium. This bubble expands, cools, and then eventually hadronizes. At this point, one assumes chemical freeze-out, the end of chemical equilibrium. The resulting interacting hadronic medium is assumed to still be in local thermal equilibrium. It continues to expand until it reaches kinetic freeze-out, leading to free hadrons. This is the end of thermal equilibrium.

The hadronization transition of the QGP is studied in finite temperature lattice QCD [10]. Close to the chiral limit of vanishing quark masses $(m_q \to 0)$, one finds critical behavior, with a correlation length $\lambda(T, m_q = 0)$ diverging at the critical point $T = T_c$,

$$\lambda(T, m_q = 0) \sim |T - T_c|^{-\nu}.$$

(15)

As a result, the correlation function become scale-invariant,

$$\Gamma(r, T_c)/\Gamma(2r, T_c) = 2^p,$$

(16)
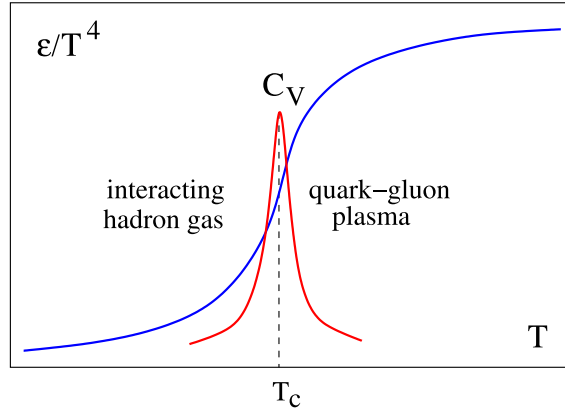
Fig. 4. Pseudo-critical behavior for the energy density ϵ and the specific heat C_v.

i.e., independent of the separation distance r.

For small finite quark masses, a reflection of criticality remains, one has "pseudo-critical" behavior at a temperature $T_c \simeq 155$ MeV (see Fig. 4).

If the local bubbles are adiabatically evolving QCD matter, i.e., if the evolution occurs in equilibrium, with a QGP above and an interacting hadron gas below T_c, then the relative hadron abundances in the hadronic medium below T_c are not fixed; they decrease as

$$\frac{\phi(m_i, T)}{\phi(m_j, T)} \sim \exp-\left(\frac{(m_i - m_j)}{T}\right). \tag{17}$$

In other words, the adiabatic evolution of a QGP does not lead to chemical freeze-out at T_c. Moreover, the hadron masses are also still temperature- dependent, as given by

$$\frac{m_h(T)}{m_h(0)} \sim \frac{f_\pi(T)}{f_\pi(0)}, \tag{18}$$

where $f_\pi(T)$ corresponds to the pion decay constant.

These features of an interacting QCD medium in equilibrium are found to disagree with high energy hadroproduction data. One there observes that the relative abundances of the different hadron species are fixed by the yields at T_c, specified by the corresponding Boltzmann factors $\phi(m_i, T_c)$, with vacuum masses m_i for the species i. In other words, an ideal gas of all hadronic resonances with vacuum masses at T_c correctly predicts "all" abundances [9]. The "all" implies some caveats.

Hadrons containing heavy flavors (charm and bottom) cannot be directly compared to those made up of light flavors, since their perturbative hard production process results in a different energy dependence than that of light flavor hadrons. In elementary collisions (e^+e^-, pp), the ideal resonance gas otherwise accounts for all hadrons, In AA collisions, it does so for stable hadrons, while resonance (ρ, K^*, N^*) can still suffer modifications due to the overlap of nucleon-nucleon interactions.

The conventional scenario thus encounters two immediate basic difficulties.
(1) Why is there chemical freeze-out directly at T_c?
(2) Why are the abundance ratios at T_c those based on vacuum masses?
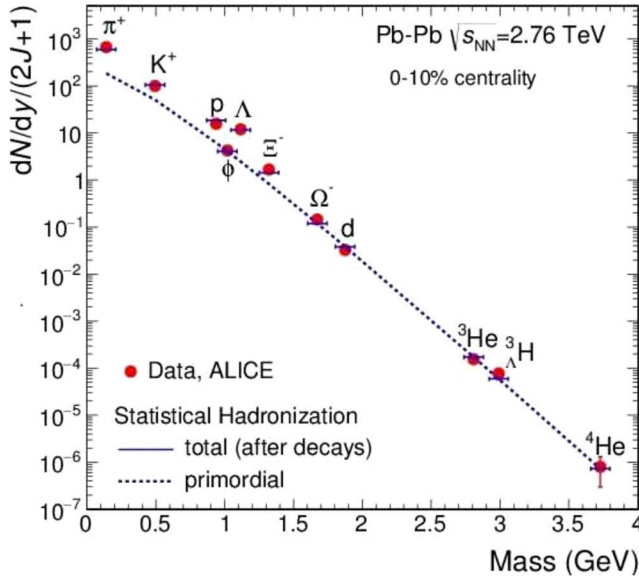
Fig. 5. Hadron yields measured at the LHC by the ALICE collaboration.

A recent further and perhaps indicative problem was triggered by recent $Pb - Pb$ LHC data by the ALICE collaboration. They find that even the abundances of light nuclei (deuteron, triton, helium) are correctly predicted by the ideal Boltzmannn gas at T_c (see Fig. 5).

These states are both large (a triton has a size of the total interaction region) and very loosely bound, so that:

(3) They cannot exist in an interacting hadronic medium of temperature T_c.

We believe that with these three points nature is trying to tell us that a scenario, in which hadronization produces an interacting hadronic medium of temperature $T \sim T_c$ and with vacuum masses, cannot be correct, and so we look for an alternative.

6 The SOC scenario

We thus consider a non-equilibrium parton system, the counterpart of pouring sand, converging towards a pseudo-critical point, at which it breaks up into all permissible hadron states – the avalanches. More specifically, in the absorptive state form of SOC, the *colored* parton state undergoes *color absorption* at the pseudo-critical point, giving rise to all possible *color neutral* hadron states.

The crucial difference to the conventional scenario of a hadronizing QGP is that in SOC the hot colored partonic medium is quenched by the cold color-neutral vacuum, breaking up into free color-neutral hadrons, without any subsequent hot interacting hadronic medium.

In SOC, the number $N(m)$ of produced hadrons of mass m, is described by the scale-invariant form

$$N(m) = \alpha[\rho(m)]^{-p}, \tag{19}$$
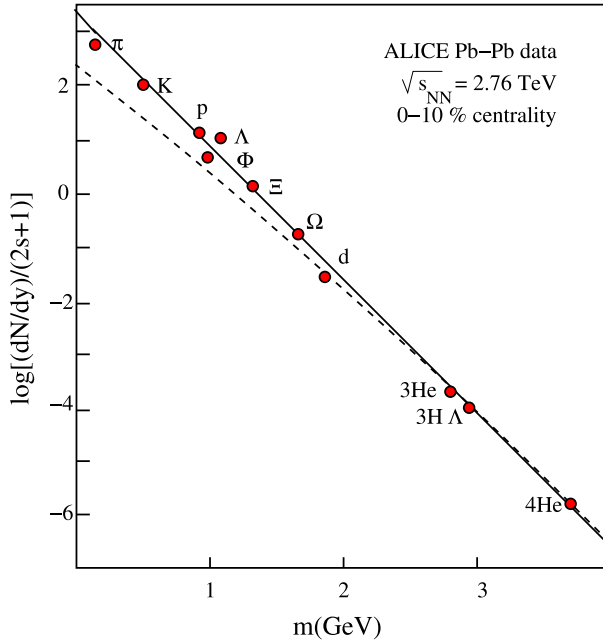
Fig. 6. ALICE data [11] compared to SOC predictions; full line equation (21), dashed line equation (20).

in terms of the resonance strength $\rho(m)$. We assume that $\rho(m)$ is given by the composition law of states in the Hagedorn bootstrap,

$$\log N(m) = -m \left(\frac{p \log e}{T_H} \right) \left[1 - \left(\frac{aT_H}{m} \right) \ln \left(1 + \frac{m}{\mu_0} \right) \right] + \text{const.} \qquad (20)$$

Let us compare the ALICE data for Pb-Pb collisions at $\sqrt{s} = 2.76$ GeV [11] to a somewhat simplified form

$$\log[(dN/dy)/(2s + 1)] \simeq -m \left(\frac{\log e \, p}{T_H} \right) + A, \qquad (21)$$

with $T_H = 155$ MeV and fit values $p = 0.9$, $A = 3.4$ (see Fig. 6).

Including the missing correction terms leads to the dashed line; the difference between the two curves accounts for the production elementary hadrons through resonance decay.

For elementary collisions, such as $p - p$, we expect the avalanches to consist of different individual hadrons. In high energy $A - A$ collisions, overlap and interactions of the "debris" can in fact affect resonance production, leading to further modifications of rates for ρ, K^*, Δ etc. The results of this are not the finite temperature equilibrium hadron gas of QCD.

7 Conclusions

We have proposed that in high energy collisions, a non-equilibrium colored parton beam converges as a function of rapidity towards a pseudo-critical attractor, the

color absorbing state. At that point, quenching leads to color neutrality in form of an avalanche of hadrons, with scale-invariant mass distributions. With evolving rapidity, there are successive avalanches, and the sum over all hadron distributions produces a thermal distribution at the pseudo-critical temperature T_c.

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