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Fertilizer and Development of an Agricultural Economy

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# Fertilizer and Development of an Agricultural Economy

## I. Fertilizer and Agricultural Production

Fertilizer is an important factor of agricultural production. Long before the use of chemical fertilizer garbarge, human and animal dung were put in agricultural production. In the rural region in Taiwan and in several other countries depots of garbarge and dung belong to the important installations. In most areas of Taiwan several agricultural products are planted as fertilizers for rice production. The use of those natural fertilizers contributes mostly to the raising of the agricultural productivity in Taiwan.

The input of chemical fertilizers raised the agricultural productivity in Taiwan revolutionally. Now the chemical fertilizers are indispensable for the agricultural production. In an earlier econometric research on rice economy in Taiwan we found out that chemical fertilizer contributes mainly to the increasing productivity of rice per hectar. [1) ] Hence chemical fertilizer is an important production factor for rice. In another paper the importance of chemical fertilizer for the other important agricultural products is observed. [2)] In a dual-econometric model for Taiwan chemical fertilizer is considered as a production factor for agricultural sector. [3)] In these econometric researches we consider chemical fertilizer as a production factor. Assuming Cobb-Douglas production function the elasticity of output with respect to input of chemical fertilizer is estimated and these estimates are statistically significantly different from zero. The ratio of determination of these production functions would be much lower if the input of chemical fertilizer were not considered. The Cobb-Douglas production function implies an unity elasticity of substitution. Our statistical tests seem to support this assumption. [4)]

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1) Chen [1974]
2) Chen [1973]
3) Chen [1972]
4) Chen [1972]

In these agricultural production functions the elasticity of output with respect to input of chemical fertilizer lies between 0.3 to 0.4 for different agricultural products and for the aggregate agricultural production.

While the importance of fertilizer for the agricultural production is well known for the agricultural technologists and the farmers, it is scarcely considered in the literature of economic development.[5)] In this paper we shall treat the fertilizer as one of the factors in a "neoclassic" agricultural production function of the Cobb-Douglas type. In the literature of economic development this type of production function is widely used.[6)] In assuming this type of production function we are especially interested to compare our findings with those of the neoclassical model of Jorgenson [1961] and the classical model of Fei and Ranis [1964].

As well as chemical fertilizers the insecticides are produced by the industrial sector and used for agricultural production. Other than chemical fertilizers the insecticides are not considered as a factor in the agricultural production function, since insecticides are used to avoid loss of agricultural production which depends mainly on the weather conditions, i.e. to reduce the influences of random factors on the agricultural production and the costs of insecticides is in general insignificant.

In this paper we shall study the development of a purely agricultural economy. The fertilizer is considered as a factor in the Cobb-Douglas agricultural production function.

In the second section a model with fertilizer produced and used by the agricultural sector will be discussed.

In the third section a model with chemical fertilizer as a factor of agricultural production is treated.

Throughout this paper we shall assume that the quantity of land in the pure agricultural economy is given fixed.

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5) Johnston and Cownie [1969]
Sahota [1968] Fertilizer in Economic Development

6) Jorgenson [1961], Fei and Ranis [1964]

We assume further that the agricultural production is organized by the farmer-families who own the land and try to maximize the final agricultural products (food-grains, for instances).

## II. Development of a pure agricultural economy with farm-produced fertilizers

Now we study a neoclassical purely agricultural economy in which fertilizers are produced by farms as intermediate product for the food production. The economy considered could be described by a modified Jorgenson-model with a fertilizer sector and explicit consideration of the fertilizer as a factor to produce food (or agricultural output for final consumption).

To compare with the finding of Jorgenson we use a modified Cobb-Douglas production function for the food outputs:

(1) $$Y = e^{\lambda t}(1+H)\, A^{\alpha}\, B^{\beta} \text{ with } \alpha,\beta > o \text{ and } \alpha+\beta \leq 1, \quad A,B \geq o$$

where Y is agricultural final output (food), A the labor used directly for food production. B is the direct input of land for food production and H is the input of farm-produced fertilizers.
The production function of the fertilizer is given by the following function

(2) $$H = C^{a}\, D^{b} \text{ [7)]} \quad \text{with } a,\ b > o \quad a + b < 1, \quad a+b+\alpha+\beta \leq 1 \text{ and } C,\ D \geq o$$

where C is the labor input and D the land input for the fertilizer production.
The elasticity of output with respect to input of fertilizer in the function (1) which is assumed to be positive and strictly less than unity is specified by the fact that the sum of both elasticities of output in function (2) is considerably less than unity.

The production function reduces to that in the Jorgenson-model if no fertilizer is used, i.e. if H = o.

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7) For simplicity we do not regard the technical progress in this function.

The input of factors for production of food and fertilizer is restricted by the fact that the labor (P) of the agricultural economy as a whole is given at a point of time and the quantity of land (L) is given fixed, i.e.

(3) $A + C \leq P$

(4) $B + D \leq L$

Really we can exclude the inequality in (3) and (4), since the marginal productivity is always positive and the food output can only be maximized if the labor and the land of the agricultural economy are fully employed.

The growth of labor which is assumed to be identic to the population is described by the following function [7]:

(5) $\hat{P} = \frac{\dot{P}}{P} = \frac{dP}{dt}\frac{1}{P} = \min\{\partial y - \delta, \varepsilon\}$

where $\hat{P}$ is the net growth rate of population, $y = \frac{Y}{P}$ is the food output per capita, $\delta$ is the constant level of force of mortality, and $\varepsilon + \delta$ is the physiological maximum.

The pure agricultural economy is completely described by the functions (1) to (5). We shall consider whether the fertilizer should be used in a static analysis and in sense of economic development.

The first point can be analysed in the static sense. The problem is to maximize the food output with or without fertilizer under the constraints of given labor force and given quantity of land.

The Lagrangian for this problem is:

$$V = e^{\lambda t}(1+C^a D^b)A^\alpha B^\beta + z_1(P-A-C)+z_2(L-B-D)$$

where $z_1$ and $z_2$ are the Lagrangian multipliers. The Kuhn-Tucker conditions can be derived easily.

7) Jorgenson [1961, p.314]

Lemma 1: The food output is maximized only if both A and B are positive.

Proof: This lemma is trivial since the food output with any positive input of A and B is strictly positive and no food output can be produced if either A or B is zero.

Q.E.D.

Lemma 2: The food output is maximized only if both labor force and land are fully employed.

Proof: This lemma is due to the fact that the marginal productivity of labor and land is always positive. Q.E.D.

From lemma 1 and 2 and the Kuhn-Tucker conditions the maximum of the food output can be chosen from the greater one of the following alternative:

(a) boundary solution:

(6) $Y^{**} = e^{\lambda t} P^{\alpha} L^{\beta}$

In this case no fertilizer is used for the food production.

(b) interior solution:

(7) $Y^{*} = e^{\lambda t}(1+H^{*}) \left[\frac{\alpha(1+H^{*})}{\alpha(1+H^{*})+aH^{*}}\right]^{\alpha} \left[\frac{\beta(1+H^{*}}{\beta(1+H^{*})+bH^{*}}\right]^{\beta} P^{\alpha} L^{\beta}$

with $H^{*} > o$

The interior solution is solved from the Kuhn-Tucker conditions with $z_1 > o$ and $z_2 > o$. Hence we have

(8) $C = \frac{aH^{*}}{\alpha(1+H^{*})} A$

(9) $D = \frac{bH^{*}}{\beta(1+H^{*})} B$

(10) $A = \frac{\alpha(1+H^{*})}{\alpha(1+H^{*})+aH^{*}} P$

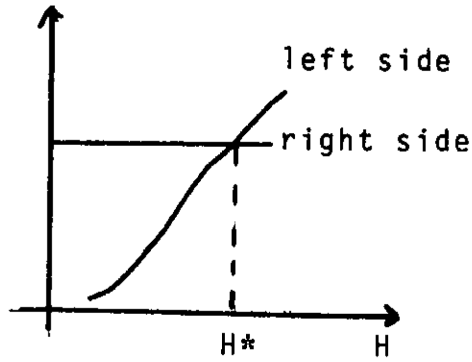
(11) $B = \frac{\beta(1+H^{*})}{\beta(1+H^{*})+bH^{*}} L$

where $H^{*}$ can be solved from the following equation:

(12) $H \left[\frac{\alpha(1+H)+aH}{\alpha(1+H)}\right]^{a} \left[\frac{\beta(1+H)+bH}{\beta(1+H)}\right]^{b} = P^{a} L^{b}$ or

(12a) $\ln H-(a+b)\ln(1+H)+a \ln[\alpha(1+H)+aH]+b \ln[\beta(1+H)+bH]$
$= a \ln\alpha P+b \ln\beta L$

Fig. 1




The solution H* can be shown in the left figure. The graph of the left side of the equation is strictly increasing. This is easily seen since its derivative is equal to

$$(13)\quad \frac{1+(1-a-b)H}{H(1+H)} + \frac{a(a+\alpha)}{\alpha(1+H)+aH} + \frac{b(b+\beta)}{\beta(1+H)+bH} > 0$$

due to the fact that $a+b<1$.

With given labor force and quantity of land at any point of time the optimal output of fertilizer is therefore uniquely determined.

Proposition 1: The maximum of the food output is an interior solution, in other words, the technology of food production with utilization of fertilizer is superior to the one without fertilizer.

Proof: This proposition can be proved by showing that $Y^* > Y^{**}$ or analogously [8)]

$$(14)\quad u = (1+H)\left[\frac{\alpha(1+H)}{\alpha(1+H)+aH}\right]^{\alpha}\left[\frac{\beta(1+H)}{\beta(1+H)+bH}\right]^{\beta} > 1$$

The condition (14) can be shown as follows:

(a) $u = 1$ for $H = 0$ and

(b) $$\frac{du}{dH} = u\{\alpha\beta\,(1+H)\,[(1+H) - a - b] + abH\,[(1+H)-\alpha-\beta] + a\beta H\,(1+H) + abH^2\} > 0$$

Hence for any $H > 0$, $u > 1$ Q.E.D.

<u>Remark</u>: The interior solution is also the global maximum of the food output.

<u>Proposition 2</u>: The use of fertilizer depends positively on the labor force and the quantity of land.
In other words, the higher the labor force or the quantity of land the more fertilizer will be used.

<u>Proof</u>: This proposition can be proved by total differentiation of (12)

$$(15) \quad \frac{\partial H^*}{\partial P} = ah\, P^{-1} > 0 \quad \text{and}$$

$$(16) \quad \frac{\partial H^*}{\partial L} = bh\, L^{-1} > 0$$

where

$$h = 1/\{ \frac{1-a-b}{H} + \frac{1+\frac{\alpha}{a}}{\frac{\alpha}{a}(1+H)+H} + \frac{1+\frac{\beta}{b}}{\frac{\beta}{b}(1+H)+H} \} > 0$$

Q.E.D.

From (8) and (9) we can compare the optimal factor intensity between the food and the fertilizer production:
$\frac{C}{D} = \frac{a}{\alpha} \frac{\beta}{b} \frac{A}{B}$ or

$$(17) \quad \frac{C}{D} : \frac{A}{B} = \frac{a}{b} : \frac{\alpha}{\beta}$$

Empirically we are convinced that the fertilizer production is more labor intensive and less land intensive than the food production. Hence $\frac{a}{b} > \frac{\alpha}{\beta}$ or $\frac{a}{\alpha} > \frac{b}{\beta}$ .

To study the development of the pure agricultural economy we calculate from (6) and (7) the following two equations of growth of per capita food production:

(18) $\hat{y}^{**} = \lambda + (\alpha-1)\hat{P}$ and

(19) $\hat{y}^{*} = \lambda + (\alpha-1) + g\hat{P}$

where

(20) $g = [\frac{1+\alpha+\beta}{1+H} - \frac{\alpha(a+\alpha)}{\alpha(1+H)+aH} - \frac{\beta(b+\beta)}{\beta(1+H)+bH}]\ Hah$

(21) $\hat{H} = ah\hat{P} > o$

Lemma 3: $1 > g > o$

Proof: To prove $g > o$ we need to show

$$\frac{1+\alpha+\beta}{1+H} - \frac{\alpha(a+\alpha)}{\alpha(1+H)+aH} - \frac{\beta(b+\beta)}{\beta(1+H)+bH} > o$$

For

$$\frac{1+\alpha+\beta}{1+H} - \frac{\alpha(a+\alpha)}{\alpha(1+H)+aH} - \frac{\beta(b+\beta)}{\beta(1+H)+bH}$$

$$= \{\alpha(1-a-b)\beta+[\alpha(1-a)(b+\beta)+\beta(1-b)(a+\alpha)]H$$

$$+ (a+\alpha)(b-\beta)H^2\} \Big/ \{(1+H)[\alpha(1+H)+aH][\beta(1+H)+bH]\} > o.$$

Since every term in the parentheses of the right hand of h is greater than unity, so that $h^{-1} > 1$ and $h < 1$.
For empirically acceptable values of H the following product is less than unity $H\left[\frac{1+\alpha+\beta}{1+H} - \frac{\alpha(\alpha+a)}{\alpha(1+H)+aH} - \frac{\beta(b+\beta)}{\beta(1+H)+bH}\right]$

Q.E.D.

Hence $1 > g > o$.

Lemma 4: $\hat{y}^{*} > \hat{y}^{**}$

Proof: This follows directly from lemma 3. Q.E.D.

Now the development of the pure agricultural economy can be described by (5) and (19). The fundamental differential equation of our model is:

(22) $\dot{y}^* = [\lambda+(\alpha+g-1)(\gamma y^* - \delta)]y^*$ if the physiological maximum is not realized, and

(23) $\dot{y}^* = [\lambda+(\alpha+g-1)\varepsilon]y^*$ if the physiological maximum is reached.

Lemma 5: The differential equation (22) has two stationary solutions:

(24) (a) $\bar{y} = \frac{\lambda+(1-\alpha-g)\delta}{(1-\alpha-g)\gamma}$

(b) $\bar{\bar{y}} = o.$

Lemma 6: The general solution of the differential equation (22) is

(25) $$y^*(t) = \bar{y} + \frac{1}{e^{[\lambda+(\alpha+g)\delta]t}\left[\frac{1}{\bar{y}} + \frac{1}{y^*(0)-\bar{y}} - \frac{1}{\bar{y}}\right]}$$

The proof for lemma 5 and 6 can be calculated directly.

Defining $y^+ = \frac{\varepsilon+\delta}{\gamma}$ for $\gamma y - \delta = \varepsilon$, $y^+$ is the per capita food output at which the physiological maximum is realized.

The development of the purely agricultural economy can be studied in three cases: 9)

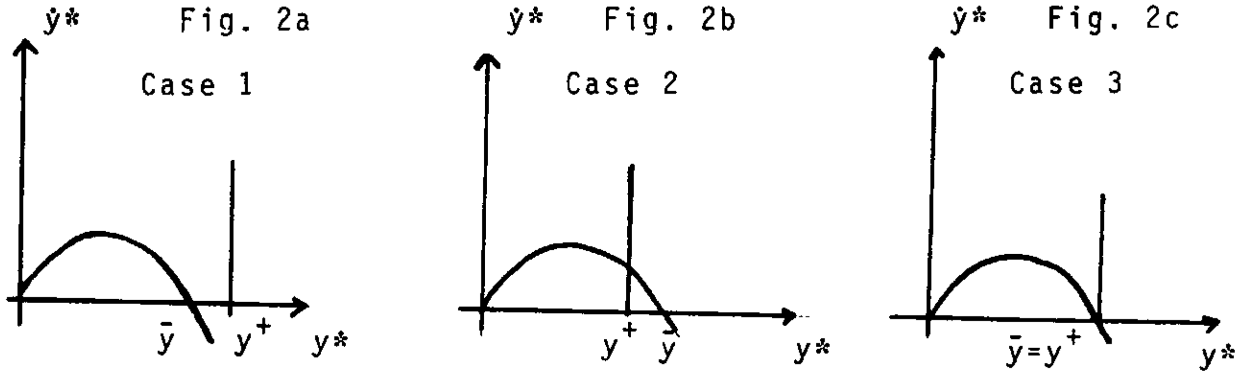
Case 1: $o < \bar{y} < y^+$ : This is the case of a "low level equilibrium trap". In this case, the per capita food production remains constant while the population grows with a rate: $\lambda/(1-\alpha-g)$. This case is characterized by

$$o < \frac{\lambda+(1-\alpha-g)\delta}{(1-\alpha-g)\gamma} < \frac{\varepsilon+\delta}{\gamma} \quad \text{or} \quad o < \frac{\lambda}{1-\alpha-g} < \varepsilon .$$

Case 2: $\bar{y} > y^+$, i.e. $\frac{\lambda+(1-\alpha-g)\delta}{(1-\alpha-g)\gamma} > \frac{\varepsilon+\delta}{\gamma}$ or $\frac{\lambda}{1-\alpha-g} > \varepsilon$ .

Case 3: $\bar{y} = y^+$, i.e. $\lambda = \varepsilon(1-\alpha-g)$.

9) See Jorgenson [1961, pp. 318-319]




These three cases can be shown by the above figures.

Proposition 3: The range of the "low level equilibrium trap" is narrowed by utilizing fertilizer for the agricultural production. The range of the low level equilibrium trap is smaller the higher g.

Proof: This main proposition can be proved by differentiating $\bar{y}$ to g:

$$\frac{\partial \bar{y}}{\partial g} = \frac{\delta \lambda}{[1-\alpha-g)\gamma]^2} > o$$

In the figure above the stationary solution $\bar{y}$ turns rightward after introducing fertilizer in the agricultural production.

Q.E.D.

Proposition 4: The long-run equilibrium growth rate of food production is higher if the technology of utilizing farm-produces fertilizer is used, or in other words, the technology of utilizing farm-produced fertilizer is a superior one in sense of economic development.

Proof: Obviously, the long-run growth rate is equal to $\lambda+(\alpha+g-1)$ as shown in (23). This is higher than the one with g = o as in the Jorgenson-model.

Q.E.D.

To show the effect of natural fertilizers on the agricultural output we calculate some numerical examples. The plausibility of the value of the parameters for our numerical examples is checked by the estimates of several econometric studies [2, 3, 8, 9, 10,11, 12, 13, 14].According to these estimates we use following values for the parameters in our model:

(a) $\lambda = 0$ (b) $\alpha = 0.4$
(c) $\beta = 0.3$ (d) $a = 0.045 - 0.27$
(e) $b = 0005 - 0.03$ (f) $a + b + \alpha + \beta \lesseqgtr 1$

The labor and the quantity of land are assumed constant equal to 10 and 24, respectively.

The numerical results are given in the following Table. Two important results of the numerical examples could be derived as follows:

Firstly, the food output can be increased considerably if natural fertilizers are used for the food production.

Secondly, the input of natural fertilizers increases the long-run equilibrium growth rate of the agricultural economy significantly.

Table: Numerical Example

| a | b | Percentage of optimal direct factor input for food production to the total factor endowment | | Effect of using natural fertilizers on the long-run equilibrium growth | Percentage of food production using natural fertilizers to that without natural fertilizers |
|---|---|---|---|---|---|
| | | land | labor | | |
| 0.04 | 0.01 | 95.3 | 98.4 | 1 | 193.9 |
| 0.045 | 0.005 | 94.7 | 99.2 | 1.1 | 195 |
| 0.08 | 0.02 | 90.7 | 96.7 | 1.9 | 194.5 |
| 0.09 | 0.01 | 89.6 | 98.3 | 2.1 | 196.8 |
| 0.12 | 0.03 | 86.3 | 95 | 2.6 | 197.7 |
| 0.135 | 0.015 | 84.6 | 97.4 | 2.9 | 201.3 |
| 0.16 | 0.04 | 81.8 | 93.1 | 3.3 | 203.0 |
| 0.18 | 0.02 | 79.7 | 96.4 | 3.6 | 208.0 |
| 0.2 | 0.05 | 77.5 | 91.2 | 3.7 | 210.1 |
| 0.225 | 0.025 | 74.9 | 95.3 | 4.1 | 216.8 |
| 0.24 | 0.06 | 73.2 | 89.1 | 4.1 | 219.2 |
| 0.27 | 0.03 | 70.3 | 94.1 | 4.5 | 227.9 |

## III. Development of a purely agricultural economy with chemical fertilizer

Since chemical fertilizer cannot be produced by the farms without an industrial sector the chemical fertilizer has to be imported. We assume that the price for chemical fertilizer ( measured in agricultural product units is given and the import of chemical fertilizer is paid by agricultural products i.e. the the balance of trade is always equalized.

To implify our presentation we shall neglect the farm-produced fertilizer. The input of chemical fertilizer is assumed to maximize the food output. The food production is described by the following function:

(26) $Y = e^{\lambda t}(1+F^a)\ P^{\alpha}\ L^{\beta} \qquad 1 > a,\alpha,\beta > o \qquad \alpha + \beta \leq 1$

where F is the input of chemical fertilizer.

Since only food will be produced the labor force and the quantity of land in the pure agricultural economy will be used for food production. If no chemical fertilizer is used, i.e. F = o, then the production function (26) is identical to the Jorgenson-model.

The input of chemical fertilizer can be described by the following function:

(27) $F = (ae^{\lambda t}p^{\alpha}L^{\beta}q^{-1})^{\frac{1}{1-a}} = sYq^{-1}, \quad s = \frac{aF^a}{1+F^a} \cong a$

where q is the terms of trade of chemical fertilizer to food. The function (27) is derived by maximizing food output with input of chemical fertilizer.

Since a part of food production is paid for chemical fertilizer only the remainder can be used for the home consumption, i.e.

(28) $Y-qF = (1-aq^{-1})\ Y = \Gamma Y, \quad \Gamma = 1-aq^{-1}$

Defining the per capita food consumption as:

(29) $x = \frac{Y-qF}{P} = (1-aq^{-1})\ y = \Gamma y, \text{when } y = \frac{Y}{P}$

From the fact that $x$ is always positive we have

$1 > aq^{-1} > o, \quad \text{i.e. } \Gamma = 1-aq^{-1} > o$ [9]

9) If this is not the case no chemical fertilizer would be used for the food production.

With the growth function of population (5) we complete our model.

Lemma 7:

$$(30)\quad Y^* = e^{\lambda t}\,[1+(ae^{\lambda t}P^{\alpha}L^{\beta}q^{-1})^{\frac{a}{1-a}}]\,P^{\alpha}L^{\beta}$$

$$(31)\quad Y^*-qF = e^{\lambda t}P^{\alpha}L^{\beta} + (ae^{\lambda t}P^{\alpha}L^{\beta}q^{-1})^{\frac{1}{1-a}}\,(e^{\lambda t}P^{\alpha}L^{\beta}-1)$$

$$(32)\quad x = \frac{Y^*-qF}{P} = e^{\lambda t}P^{\alpha-1}L^{\beta} + (ae^{\lambda t}L^{\beta}q^{-1})^{\frac{1}{1-a}}\,P^{\frac{a+\alpha-1}{1-a}}\,(e^{\lambda t}P^{\alpha}L^{\beta}-1)$$

Proof: Set (27) in (26) we get (30).
Using (30) and (27) we have (31)
Devising (31) by P we get (32). Q.E.D.

Proposition 5: The food production utilizing chemical fertilizer is advantageous, i.e. the net food production after paying the input of chemical fertilizer is greater than the food production without utilizing of chemical fertilizer.

Proof: To prove this proposition we shall show that

$$(33)\quad Y^* - qF - e^{\lambda t}P^{\alpha}L^{\beta} > 0$$

Made use of (31) we have

$$Y^*-qF-e^{\lambda t}P^{\alpha}L^{\beta} = (ae^{\lambda t}P^{\alpha}L^{\beta}q^{-1})^{\frac{1}{1-a}}\,(e^{\lambda t}P^{\alpha}L^{\beta}-1)$$

Hence $Y^*-qF-e^{\lambda t}P^{\alpha}L^{\beta} > 0$ if $e^{\lambda t}P^{\alpha}L^{\beta} > 1$.
Since $\lambda + \alpha\hat{P} > 0$, this condition will become true even if it could not be fulfilled in the beginning period of the development of the purely agricultural economy considered. Q.E.D.

To study the development of the purely agricultural economy using technology with chemical fertilizer we have

$$(34)\quad \hat{Y} = \lambda + \alpha\,\hat{P} + s\,\hat{F} \quad \text{where } s = \frac{aF^{a}}{1+F^{a}} < a$$

the growth equation of production function (26) and

$$(35)\quad \hat{F} = \hat{Y}$$

the growth equation of the input of chemical fertilizer (27).

Set (35) in (34) we have

(36) $\hat{Y} = \frac{\lambda}{1-s} + \frac{\alpha}{1-s}\hat{P}$ and

(37) $\hat{y} = \hat{y} - \hat{P} = \frac{\lambda}{1-s} + \frac{\alpha+s-1}{1-s}\hat{P}$

Made use of (5) and (29) the growth of population for the case that the physiological maximum is not reached can be given by:

(38) $\hat{P} = \Gamma y - \delta > \gamma e^{\lambda t} P^{\alpha-1} L^{\beta} - \delta$ (Proposition 4).

Substituting (38) in (37) we have the fundamental differential function for the system of a pure agricultural economy in which chemical fertilizer is put in the production:

(39) $\dot{y} = (w_1 + w_2\delta - w_3\gamma y)y$

when $w_1 = \frac{\lambda}{1-s} > \lambda$ , $w_2 = 1 - \frac{\alpha}{1-s} < 1-\alpha$ , $w_3 = w_2\Gamma < w_2$

The single meaningful stationary solution for the fundamental differential equation is:

(40) $\bar{y} = \frac{w_1 + w_2\delta}{w_3\gamma} = \frac{w_1}{w_3\gamma} + \frac{\delta}{\Gamma\gamma}$

As in the model studied in the last section three different cases of growth of the pure agricultural economy can be observed, i.e. (a) $\bar{y} < y^+$, (b) $\bar{y} > y^+$ and (c) $\bar{y} = y^+$, where is the per capita net food production (After paying the input of chemical fertilizer) for the physiological maximum. The "low-level equilibrium trap" can only exist in the case (a) [10]

Proposition 6: The realization of a low-level equilibrium trap is less probable if chemical fertilizer is put in the food production or, in other words, the range of a low-level equilibrium trap is narrowed by the input of chemical fertilizer in the food production.

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10) See Fig. 2a, 2b and 2c.

Proof: To prove this proposition we need to show that the stationary solution $\bar{y}$ is higher than the stationary solution in the Jorgenson-model in which no chemical fertilizer is considered.

In the Jorgenson-model s = o and $\Gamma = 1$. If chemical fertilizer is put in the food production, as we have shown above, then $1 > s > o$ and $1 > \Gamma > o$. Obviously, the stationary solution in our model ($\bar{y}$) is higher than the one in the Jorgenson-model. Q.E.D.

Proposition 7: The growth rate of food production is higher if chemical fertilizer is used in the food production, i.e. the technology with utilizing of chemical fertilizer is a superior one in sense of economic development.

Proof: The case of a long-run equilibrium growth in our model can be described by the following differential equation:

$$\dot{y} = (w_1 - w_2\epsilon)y$$

with a long-run equilibrium growth rate of $w_1 - w_2\epsilon$.

In the Jorgenson-model the long-run equilibrium growth rate is equal to "$\lambda-(1-\alpha)\epsilon$" (shown in symbols of our model).

Obviously $w_1 - w_2\epsilon > \lambda - (1-\alpha)\epsilon$ since $w_1 > \lambda$ and $w_2 < (1-\alpha)$. Q.E.D.

## IV. Summary:

In this paper we study the meaning of fertilizers, both farm-produced and chemical fertilizers, for the development of a purely agricultural economy. The Jorgenson-model is modified by introducing fertilizer as one of the production factor in the food production. Throughout this paper our presentation is undertaken so that the results of our models can be easily compared to the corresponding ones of the Jorgenson-model.

In several earlier studies on the agricultural economy of Taiwan we found out that chemical fertilizer is one of the most important production factor. Fertilizer can also be produced by farms using labor and land as intermediate product for the food production. Really, both farm-produced and chemical fertilizer are used for agricultural production. In the literature of economic development hitherto the meaning of fertilizer is scarcely considered.

Important findings of this study can be summarized as follows:

(i) the food production (the agricultural products for consumption) can be increased considerably if fertilizers, both farm-produced and chemical, are used.

(ii) The realization of a low-level equilibrium trap in a pure agricultural economy is remarkably less probable, i.e. the range of the low-level equilibrium trap is remarbably narrowd if fertilizers, both farm-produced and chemical, are put in the food production; and

(iii) the long-run equilibrium growth rate of the purely agricultural economy considered will be increased considerably if either farm-produced or chemical fertilizer is used for the food production.

Hence the technology of utilizing fertilizers is a superior one both in static sense and in sense of economic development. The scene of the economic development of the underdeveloped economies is much optimistic if the technology of utilizing fertilizers would be introduced in the economies.

In a classical model the meaning of fertilizers for the economic development can be studied analogously.

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