Thermal fluctuations of Chern-Simons numbers in the lattice SU(2) Higgs model

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We study the temperature dependence of the Chern-Simons number fluctuations in the SU(2) Higgs model on Euclidean lattices with spatial sizes up to 20^3. Temperatures well above the Higgs phase transition T_H are achieved on anisotropic lattices. Numerical results are compared to perturbative results on finite lattices as well as in continuum perturbation theory. We find qualitative agreement with perturbative estimates and see at high temperatures a tendency towards static configurations. Up to temperatures $T \simeq 2T_H$ we find an indication that tunneling between vacuums with different Chern-Simons numbers is still exponentially suppressed.

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I. INTRODUCTION

The nonconservation of the baryon and lepton numbers in the electroweak theory is well known [1]. It has been argued that the baryon-number-violating processes can be strongly enhanced at high temperature. Calculations of the corresponding rates are based on the one hand on semiclassical estimates for transitions between topologically distinct vacuums of the electroweak theory [2] and, on the other hand, they have been performed through Monte Carlo simulations within the framework of an effective Hamiltonian model, which is derived from the finite temperature Euclidean theory in the high temperature limit [3]. Both approaches rely on dimensional reduction, which is expected to be valid at high temperature and should allow us to treat the timelike component of the SU(2) gauge fields as static fields. In the vicinity of the electroweak phase transition this approach breaks down and a nonperturbative understanding of the structure of the topologically distinct vacuums and their temperature dependence will be important. We will study here thermal fluctuations of Chern-Simons number distributions within the framework of the Euclidean formulation of the SU(2) Higgs model on the lattice. From an analysis of their correlation in Euclidean time, we will be able to test the validity of the static approximation in the vicinity of the electroweak phase transition. The nonperturbative results for this correlation as well as moments of the Chern-Simons numbers will be compared with high temperature perturbation theory in the continuum and on the lattice.

A baryon-number-violating process is a transition from a vacuum field configuration to one in another vacuum, which cannot be reached through small gauge transformations from the initial one. In the $A_0=0$ gauge it is possible to assign a Chern-Simons number

$$n_{\rm CS}=-\frac{1}{8\pi^2}\int d^3x\, \epsilon_{ijk}{\rm tr}[A_i(\partial_j A_k+\tfrac{2}{3}A_j A_k)] \quad (1.1)$$

to gauge field configurations at some fixed time. Vacuums always have integer $n_{\rm CS}$, and the change in baryon number during the transition between two of them is proportional to the difference of their Chern-Simons numbers, the proportionality constant being the number of quark and lepton families. The rate of transitions between different vacuum sectors depends on the height of the potential barrier between them. For temperatures below the electroweak phase transition, the rate for baryon-number-violating processes can be estimated, making use of the existence of classical field configurations, *sphalerons,* which interpolate between two vacuums and carry half-integer Chern-Simons numbers. This approximation breaks down close to T_c. For temperatures much larger than T_c, baryon-number-changing processes are expected to be frequent, $\sim T^4$; however, no accurate estimate exists. In particular, in the vicinity of T_c the transition rates in the high temperature phase are unknown. A nonperturbative study of the different topological vacuums in the electroweak theory is thus needed.

In general the barrier height between vacuums with different Chern-Simons numbers can be determined from the difference of the Chern-Simons effective potential $V(n_{\rm CS})$ at $n_{\rm CS}=0$ and $n_{\rm CS}=\frac{1}{2}$, which is defined as

$$P(n_{\rm CS})=e^{V(n_{\rm CS})}=\int[DA][D\phi]e^{-S}\delta\left[n_{\rm CS}+\frac{1}{8\pi^2}\int d^3x\,\epsilon_{ijk}{\rm tr}[A_i(\partial_j A_k+\tfrac{2}{3}A_j A_k)]\right].$$

Here $P(n_{\rm CS})$ is the probability distribution of the Chern-Simons numbers. The shape of the effective potential has been studied in the semiclassical approach [4]. Having a lattice prescription for the evaluation [5,6] of $P(n_{\rm CS})$ at hand, one can extract the barrier height nonperturbatively also. The temperature dependence of the barrier height can be used to determine the temperature dependence of the transition rate, up to a normalization factor tions between Chern-Simons numbers on different (Euclidean) time slices. The basic assumption in analytic continuum calculations, as well as real time simulations on the lattice [3,8], is that at high temperature the relevant field configurations become static. This should be reflected in an increasing correlation between Chern-Simons numbers on neighboring time slices. The perturbative calculation yields

$$\langle n_{\rm cs}(0) n_{\rm CS}(t)\rangle = \frac{6\xi^2}{N_\tau^2}\left[\frac{g^2}{32\pi^2}\right]^2 \sum_{\vec{p},p_0,q_0}{}' \frac{S_{\vec{p}}^2 \cos(q_0 t)}{[S_{\vec{p}}^2+\xi^2\sin^2(p_0/2)][S_{\vec{p}}^2+\xi^2\sin^2(q_0/2)]} . \tag{3.8}$$

In the following we will use these perturbative expressions for comparison with our Monte Carlo data. This will allow us to judge how far the thermal fluctuations in the Chern-Simons numbers show perturbative behavior in the temperature and coupling regime studied by us.

IV. NUMERICAL RESULTS

All our simulations have been performed with the set of couplings $\beta=4/g^2=8$, $\kappa=0.129\,96$, and $\lambda=0.001\,723\,5$ [9], and various values of the anisotropy $\xi\in[1,2.5]$. The finite temperature simulations have generally been performed on lattices of size $N_\tau \times N_\sigma^3$ with $N_\tau=2$ and N_σ ranging from 4 to 20. Some simulations have been performed on lattices with larger values of N_τ in order to check that within our statistical accuracy our finite temperature results only depend on the ratio $a_\sigma T=\xi/N_\tau$. For the zero temperature subtractions we use symmetric lattices of size N_σ^4 with the same value of ξ. We note that we will calculate only the gauge invariant noninteger part of $n_{\rm CS}$, which is normalized such that $n_{\rm CS}\in[-\frac{1}{2},\frac{1}{2}]$. In the case of a flat distribution of Chern-Simons numbers, which will be reached in the limit of large volumes or temperatures, one will thus find the limiting value

$$\langle n_{\rm CS}^2\rangle_\xi(N_\sigma,N_\tau) \xrightarrow[N_\sigma,\xi/N_\tau\to\infty]{} \frac{1}{12} . \tag{4.1}$$

This constrains the range of useful lattice sizes for our simulations. However, it also indicates that the perturbative calculations are only valid for small values of LT. Taking the perturbative results as a first guidance, we find that for $g^2=0.5$, as we will use it in our simulations,[2] the asymptotic value $\langle n_{\rm CS}^2\rangle=\frac{1}{12}$ will be reached for $LT=N_\sigma\xi/N_\tau\simeq 20$. As will become clear from the following discussion, we indeed find that the limit given by Eq. (4.1) is reached for $LT\simeq 10$, and thus for $N_\tau=2$ we can work on rather large spatial lattices up to size $N_\sigma=20$.

[2]The small value of g^2 used in this study has an additional advantage over our earlier investigation [6]: Because of the smoothness of the gauge fields the evaluation of Chern-Simons numbers by numerical integration is much simplified. This also improves the speed of our numerical algorithm, which to a large extent vectorizes and on a Cray-YMP or NEC-SX-3 typically yields one Chern-Simons number in 40 sec on a 2×10^3 lattice.

Some distributions of Chern-Simons numbers on isotropic lattices ($\xi=1$) are shown in the first two rows of Fig. 2. We note that with increasing spatial lattice size the distributions get significantly broader. However, as expected we also find that in this regime of couplings large values of $n_{\rm CS}$, in the vicinity of $n_{\rm CS}=\frac{1}{2}$, are strongly suppressed at zero temperature and even close to T_c (second row in Fig. 2), although the somewhat broader distributions found in this latter case suggest an increasing tunneling probability compared to that at zero temperature.

In Fig. 3 we show some results for the width of the Chern-Simons number distributions, without performing any vacuum subtractions. The approach to the limiting value of $\frac{1}{12}$ on large lattices is clearly visible. It is also obvious that on smaller lattices the fluctuations grow linear-

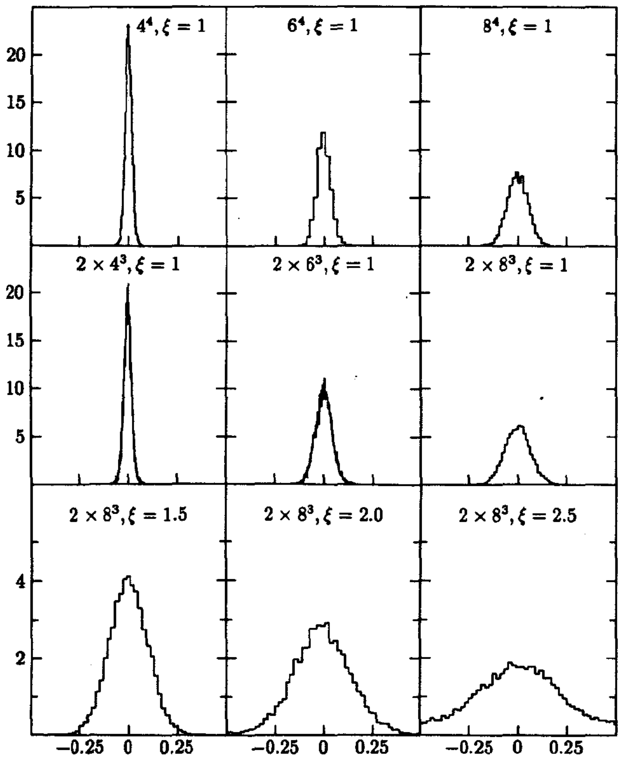

FIG. 2. Chern-Simons number distributions on various lattices and different anisotropies ξ as indicated. The top row gives $T=0$ distributions at growing spatial volume. There are only small differences at $T=T_c$ (middle row). With increasing temperature at lattices of fixed spatial size the distributions get significantly broader (lowest row).

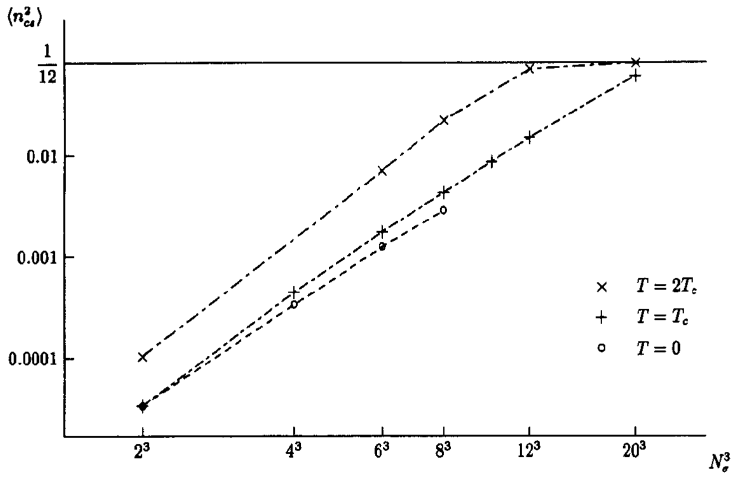

FIG. 3. Width of the Chern-Simons number distributions as a function of spatial volume.

ly with volume. In fact, we find that the first two nonvanishing moments of n_{CS} grow proportionally to the spatial volume and its square, respectively. For $N_\sigma \le 20$ one obtains

$$\langle n_{\text{CS}}^2 \rangle_{\xi=1}(N_\sigma, N_\tau) = \begin{cases} (5.9 \pm 0.5) \times 10^{-6} N_\sigma^3, & N_\tau = N_\sigma, \\ (8.5 \pm 0.2) \times 10^{-6} N_\sigma^3, & N_\tau = 2, \end{cases}$$

$$\langle n_{\text{CS}}^4 \rangle_{\xi=1}(N_\sigma, N_\tau) = \begin{cases} (9.6 \pm 0.6) \times 10^{-11} N_\sigma^6, & N_\tau = N_\sigma, \\ (17.0 \pm 0.6) \times 10^{-11} N_\sigma^6, & N_\tau = 2. \end{cases} \tag{4.2}$$

Details on the statistics and the results for all the moments measured by us on various lattices are given in Table I. The small change of the distribution as a function of temperature below T_c is also confirmed by our simulations on symmetric lattices of size 8^4 with anisotropic couplings $\xi = 2$ and 4, which corresponds to temperatures $T/T_c \simeq 0.5$ and 1.0.

In order to obtain information at even higher temperature in the symmetric phase, we used anisotropic lattices with temporal extent $N_\tau = 2$. Some results for anisotropy $\xi = 1.5$, 2, and 2.5, corresponding to $T/T_c = 1.5$, 2, and 2.5, are shown in the last row of Fig. 2. The rapid broadening of the distributions with increasing ξ is clearly visible. The resulting width of the distribution for anisotropy $\xi = 2$, corresponding to $T \simeq 2T_c$, is also given in Fig. 3 together with results for $T \simeq 0$ and T_c. For $\xi = 2$ we reach the limit of a flat distribution, resulting in $\langle n_{\text{CS}}^2 \rangle_\xi = \frac{1}{12}$, for $N_\sigma \simeq 12$, i.e., $LT = 12$. Our numerical simulations thus have to stay in the regime with $LT \le 10$. As discussed in the previous section, we should compare our numerical data in this parameter regime with perturbation theory in a finite volume.

A systematic analysis of the temperature dependence of the width of the distributions has been performed on 2×8^3 lattices with anisotropies varying between $\xi = 1.0$ and $\xi = 2.5$. With increasing temperature the distributions become rapidly broader. Results for the thermal part of the width, calculated according to the prescrip-

TABLE I. The first two nonvanishing moments of Chern-Simons number distributions on lattices of size $N_\tau \times N_\sigma^3$ and anisotropy ξ. Also given is the number of configurations analyzed for each set of parameters. All gauge field configurations are separated by ten sweeps of overrelaxed heat bath updates.

N_σ	N_τ	ξ	No.	$\langle n_{\text{CS}}^2 \rangle$	$\langle n_{\text{CS}}^4 \rangle$
2	2	1.0	8000	0.000 003(2)	0.000 000 004 6(04)
2	2	2.0	8000	0.000 104(5)	0.000 000 042 1(30)
4	4	1.0	8400	0.000 334(10)	0.000 000 35(2)
4	2	1.0	9840	0.000 443(15)	0.000 000 62(3)
4	2	1.2	8000	0.000 622(20)	0.000 001 22(6)
4	2	1.5	1200	0.000 966(50)	0.000 002 9(4)
4	2	1.6	8000	0.001 016(30)	0.000 003 4(2)
4	2	1.7	8000	0.001 132(30)	0.000 004 1(2)
4	2	1.8	8000	0.001 294(30)	0.000 005 6(3)
4	2	2.0	1000	0.001 73(20)	0.000 008 75(150)
4	2	2.2	6628	0.002 021(50)	0.000 013 6(10)
4	2	2.4	6848	0.002 399(60)	0.000 019 1(15)
4	2	3.0	1000	0.004 80(14)	0.000 086 7(80)
6	6	1.0	1560	0.001 23(7)	0.000 004 9(005)
6	2	1.0	7680	0.001 75(6)	0.000 009 4(005)
6	2	1.2	4036	0.002 42(7)	0.000 018 9(12)
6	2	1.5	2058	0.003 8(2)	0.000 044 7(35)
6	2	1.8	4160	0.005 4(2)	0.000 094 6(90)
6	2	2.0	7760	0.007 0(3)	0.000 155(10)
6	2	2.2	6914	0.010 1(4)	0.003 70(40)
6	2	2.5	5472	0.012 8(4)	0.005 04(30)
8	8	1.0	2400	0.002 84(13)	0.000 024(2)
8	8	2.0	7840	0.004 68(15)	0.000 065(4)
8	8	4.0	2952	0.003 20(17)	0.000 032(4)
8	2	1.0	9120	0.004 26(13)	0.000 054(4)
8	2	1.3	2004	0.007 31(60)	0.000 169(20)
8	2	1.5	9924	0.009 76(40)	0.000 293(20)
8	2	1.8	3444	0.015 39(110)	0.000 729(70)
8	2	2.0	7732	0.022 41(100)	0.001 62(20)
8	2	2.5	18564	0.046 21(200)	0.005 45(30)
10	2	1.0	2720	0.008 64(50)	0.000 23(2)
12	2	1.0	1860	0.015 1(11)	0.000 65(7)
12	2	2.0	1830	0.072 1(55)	0.010 3(7)
20	2	1.0	1352	0.061 8(40)	0.008 3(5)
20	2	2.0	1156	0.083 0(50)	0.012 4(5)

tion given in Eq. (3.7), are shown in Fig. 4 together with the perturbative result. The width clearly rises rapidly with ξ. The double logarithmic plot nicely shows a power law behavior $\langle n_{CS}^2 \rangle \sim \xi^\alpha$. The best fit gives a power of $\alpha = 3.7(1)$, which should be compared to the temperature dependence $\langle n_{CS}^2 \rangle \sim T^3$ expected from continuum perturbation theory. Here one has to take into account that we have ignored quantum corrections in the lattice anisotropy couplings $\gamma_{g,h}$, which modify the tree level relation $\gamma_{g,h} = \xi \sim T$ between these couplings and the temperature. For instance, assuming for the $O(g^2)$ correction to the anisotropic couplings the form $\gamma_{g,h} = \xi[1 + cg^2(\xi - 1)]$, which is valid for ξ close to 1, will reduce the power to 3.2(1) for $cg^2 = 0.1$ and 2.9(1) for $cg^2 = 0.2$. We thus consider our result for the ξ dependence of the width as rather satisfactory.

In addition to this agreement in the functional form of the temperature dependence we find, however, that the thermal width of the Chern-Simons number distributions generally is about a factor 3.0 larger than the perturbative value. For $\xi \geq 1.8$ we also find a statistically significant probability for half-integer Chern-Simons numbers. This allows us to attempt an estimate of the temperature dependence of the tunneling rate between different Chern-Simons vacuums. Assuming that the number of tunneling events is proportional to the number of configurations with Chern-Simons numbers close to half-integer values, i.e., $n_{CS} \in [n + \frac{1}{2} - \varepsilon, n + \frac{1}{2} + \varepsilon]$, $n \in Z$, we can compare the ratios of tunneling rates at different temperatures (Table II).

We define $F(\varepsilon, \xi)$ as the fraction of configurations on lattices with anisotropy ξ (temperature $a_\sigma T = \xi / N_\tau$) for which the absolute value of n_{CS} differs by less than ε from $\frac{1}{2}$. At zero temperature we do not find any configurations with Chern-Simons numbers close to $\frac{1}{2}$. We thus can directly use the results obtained on the $N_\tau = 2$ lattices to determine $F(\varepsilon, \xi)$. The data are given in Table II. From this we can eliminate the unknown proportionality constant, which relates F to the tunneling rate Γ.

As long as the temperature is much smaller than the W-boson mass the latter determines the scale for finite en-

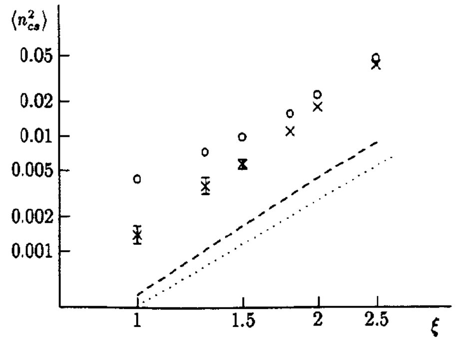

FIG. 4. The temperature dependence of the width on $8^3 \times 2$ lattices with anisotropies ξ ranging from 1.0 up to 2.5 (○). Subtraction of the appropriate zero temperature contribution gives the values (×). Errors are plotted only if they are bigger than the symbol. The dashed and dotted curves give the perturbative results in the continuum and on the lattice, respectively.

TABLE II. The first table gives the fraction of Chern-Simons numbers, $F(\varepsilon, \xi)$, calculated on 2×8^3 lattices with anisotropy ξ which differ from half-integer values by less than ε. In the second table we compare some ratios $F(\varepsilon, \xi_1)/F(\varepsilon, \xi_2)$ with the semiclassical estimate, Eq. (4.3), for the tunneling rate in the two limiting cases $m_W = m_W(T=0)$ (a) and $m_W \sim T$ (b), respectively.

ξ	$F(0.1, \xi)$	$F(0.05, \xi)$
1.8	0.0035(13)	0.0013(07)
2.0	0.0109(23)	0.0044(11)
2.5	0.0710(45)	0.0327(28)

ξ_1/ξ_2	$\varepsilon = 0.1$	$\varepsilon = 0.05$)	(a)	(b)
2.0/1.8	3.1±1.3	3.4±2.0	3.9	1.5
2.5/2.0	6.5±1.4	7.4±2.0	10.0	2.4

ergy sphaleron solutions which enter the semiclassical estimates of the tunneling rate [2]. One finds exponentially small tunneling rates per unit time and volume:

$$\Gamma / tV = 0.007(\alpha_W T)^4 \left[\frac{3M_W}{T\alpha_W} \right]^7 e^{-3M_W / T\alpha_W} . \quad (4.3)$$

At high temperatures the relevant energy scales are $\sim T$, which should lead to a replacement of m_W by a term $\sim T$ in the above estimate. One thus would expect that in the high temperature limit the tunneling rates become proportional to T^4 [2,8]. In order to compare the temperature dependence of the tunneling rates found in our numerical simulation with the above semiclassical relation, we determine the temperature and the coupling α_W from our simulation parameters as $T = \xi / N_\tau a_\sigma$ and $\alpha_W = 1/\beta\pi$, respectively. For the mass scale, m_W we consider the two extreme cases $m_W \sim T$ and $m_W = m_W(T=0)$, where $a_\sigma M_W(T=0) = 0.2$ is taken in accordance with the Monte Carlo simulation of Ref. [9]. As can be seen from Table II, in the temperature regime studied by us the data seem to favor a mass scale which is still only weakly temperature dependent. This is, in fact, consistent with the findings of Ref. [9], where little temperature dependence has been observed for the W-boson mass across the phase transition, while the Higgs boson mass dropped significantly close to T_H. Because of the low statistics the errors on our numerical results for the tunneling rates are still quite large. However, it is reassuring that the results do not seem to depend much on the value chosen for ε.

Finally, we want to test to what extent the static approximation used in analytical approaches is supported by our four-dimensional simulations. Static configurations should display strong correlations between the Chern-Simons numbers calculated on neighboring time slices. This is easily visualized if the Chern-Simons numbers calculated on the two time slices of our $N_\tau = 2$ lattices are plotted against each other (Fig. 5). At low temperatures the two numbers are uncorrelated, resulting in a spherical distribution in the scatter plot. With increasing temperature, however, the two measurements get more and more correlated.

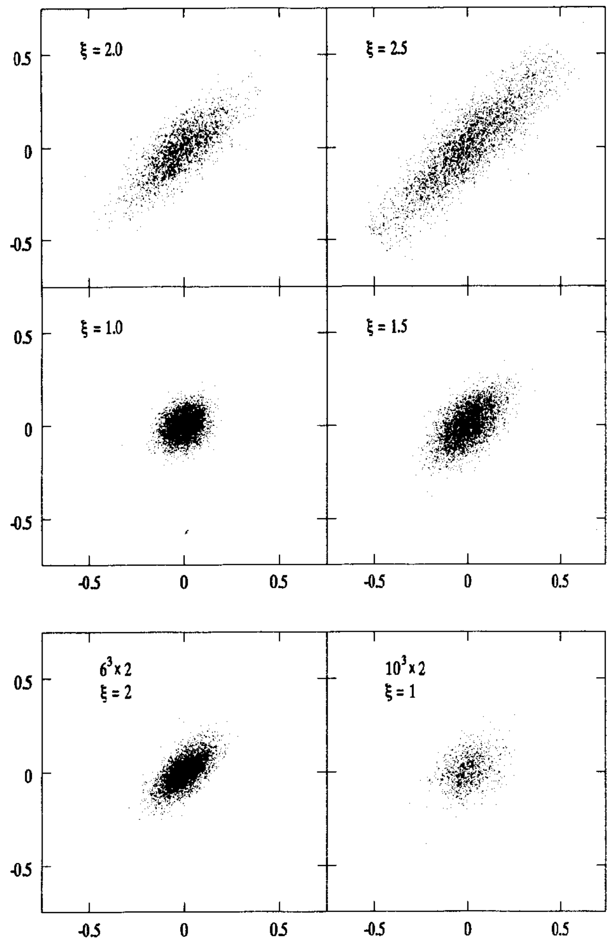

FIG. 5. The scattering of Chern-Simons numbers for the two time slices is given for fixed lattice and increasing temperature (top). Below, the numbers are displayed for simulations with similar width but different temperature.

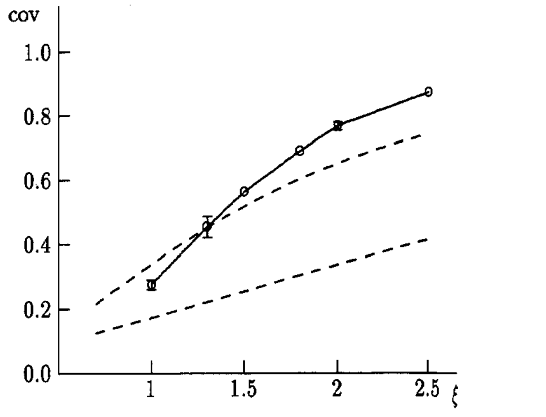

FIG. 6. The correlation between the two Chern-Simons numbers on the 2×8^3 lattices versus anisotropy ξ. As a measure for it we used the covariance. The dashed curves give the perturbative values on the lattices with $N_\tau=2$ (top) and $N_\tau=N_\sigma$ (bottom), respectively. Error bars are only drawn if they are bigger than the symbol.

The same behavior appears when comparing the results from simulations on different lattices which yield a similar width but correspond to different temperatures, e.g., $6^3\times 2, \xi=2$ and $8^3\times 2, \xi=1$ (Fig. 5). The strength of these correlations can be measured by the covariance matrix

$$\mathrm{cov}=(\langle xy\rangle-\langle x\rangle\langle y\rangle)/\sqrt{(\langle x^2\rangle-\langle x\rangle^2)(\langle y^2\rangle-\langle y\rangle^2)}\,,$$

with x,y denoting the two Chern-Simons numbers. Because of the periodic structure of the Chern-Simons term some care has to be taken when projecting two Chern-Simons numbers simultaneously to the restricted interval $[-\frac{1}{2},\frac{1}{2}]$, as artificial correlations can build up. On the other hand, existing correlations may also be destroyed because pairs of the form $(0.5+\varepsilon, 0.5-\varepsilon)$ become separated after projection. Therefore the numbers were shifted in pairs, minimizing the distance from (0,0). Taking this definition, there is a clear indication for a growing correlation with increasing temperature (Fig. 6).

V. CONCLUSIONS

We have studied the temperature dependence of Chern-Simons number distributions on Euclidean lattices. In our parameter range we were able to produce statistically significant distributions of Chern-Simons numbers, which clearly showed the expected broadening of the distributions with increasing temperature.

A comparison with perturbative calculations on the lattice as well as in the continuum shows that the width of these distributions typically is about a factor of 3 larger than expected from perturbation theory. For temperatures $T\geq 1.8T_H$, we find statistically significant fractions of configurations with Chern-Simons numbers close to $\pm\frac{1}{2}$. These configurations have been related to the number of tunnelings between topologically distinct vacuums. We find that the corresponding tunneling rates are still controlled by an energy scale consistent with that of the zero temperature W-boson mass. The rates do, however, start growing rapidly between $T=T_H$ and $T=2.5T_H$, while they show little temperature dependence below T_H, as can be deduced from the small changes in the first two nonvanishing moments of the Chern-Simons number distributions.

Our present analysis is limited to a temperature and volume range given by the constraint $LT<10$, which essentially is dictated by the occurrence of large contributions from vacuum fluctuations. If we want to reach even higher temperatures on larger lattices, we have to perform simulations at smaller values of the gauge coupling. With our present algorithms this should be feasible and it should then be possible to perform a systematic study of the temperature dependence of the tunneling rates over a wide temperature regime. It would certainly be interesting to check at which temperatures one reaches a regime where the asymptotically expected scaling of the transition rate with the fourth power of the temperature is valid.

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